

Parabolas – Graphs of Quadratic Functions

Vocabulary:

1. **Quadratic Function** – A function of the form: $y = ax^2 + bx + c$. *Standard Form.*
2. **Coefficients** – The “a”, “b” and “c” in a quadratic expression. They are typically real number values.
3. **Parabola** – The name given to the graph of a quadratic function.

Problem 1 – “What U Talkin Bout?”

Consider the quadratic function: $y = x^2 + 4x + 3$.

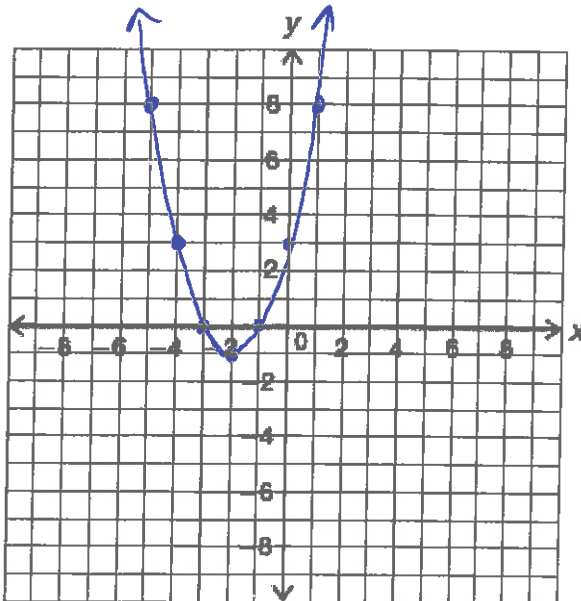
1. Determine the values of the coefficients.

$$a = 1$$

$$b = 4$$

$$c = 3$$

2. Graph the quadratic using your calculator. Use a standard window $[-10, 10] \times [-10, 10]$. Sketch the graph on the grid below.



- a. Describe the characteristic shape of the graph.

the graph is a "U" shape.

- b. What is the y-intercept of the graph?

(0, 3)

- c. What are the x-intercepts of the graph?

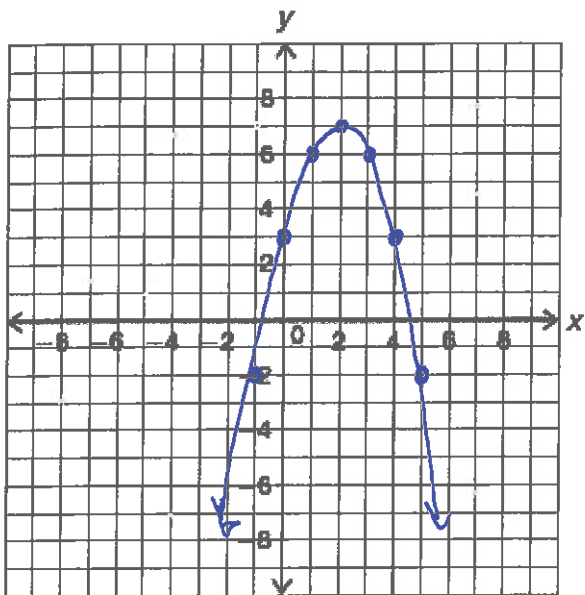
(-3, 0), (-1, 0)

Problem 2 – “Leading the Way”

The value “a” in a quadratic expression is referred to as the **Leading Coefficient**.

Changing the sign of the leading coefficient will alter the graph of the quadratic.

1. Change the sign of the leading coefficient of $y = x^2 + 4x + 3$. Sketch the resulting graph below.



- a. What change to the graph did you observe?

graph is opening down rather than opening up.

- b. Describe the characteristic shape of the graph of a quadratic that has a **positive** leading coefficient.

opens up.

- c. Describe the characteristic shape of the graph of a quadratic that has a **negative** leading coefficient.

opens down.

2. Determine if the graph of the quadratic is opening up or down by analyzing the leading coefficient.

a. $y = 4x^2 - 3x + 1$

up.

b. $y = -2x^2 + 7$

down

c. $y = -5x^2 - 2x$

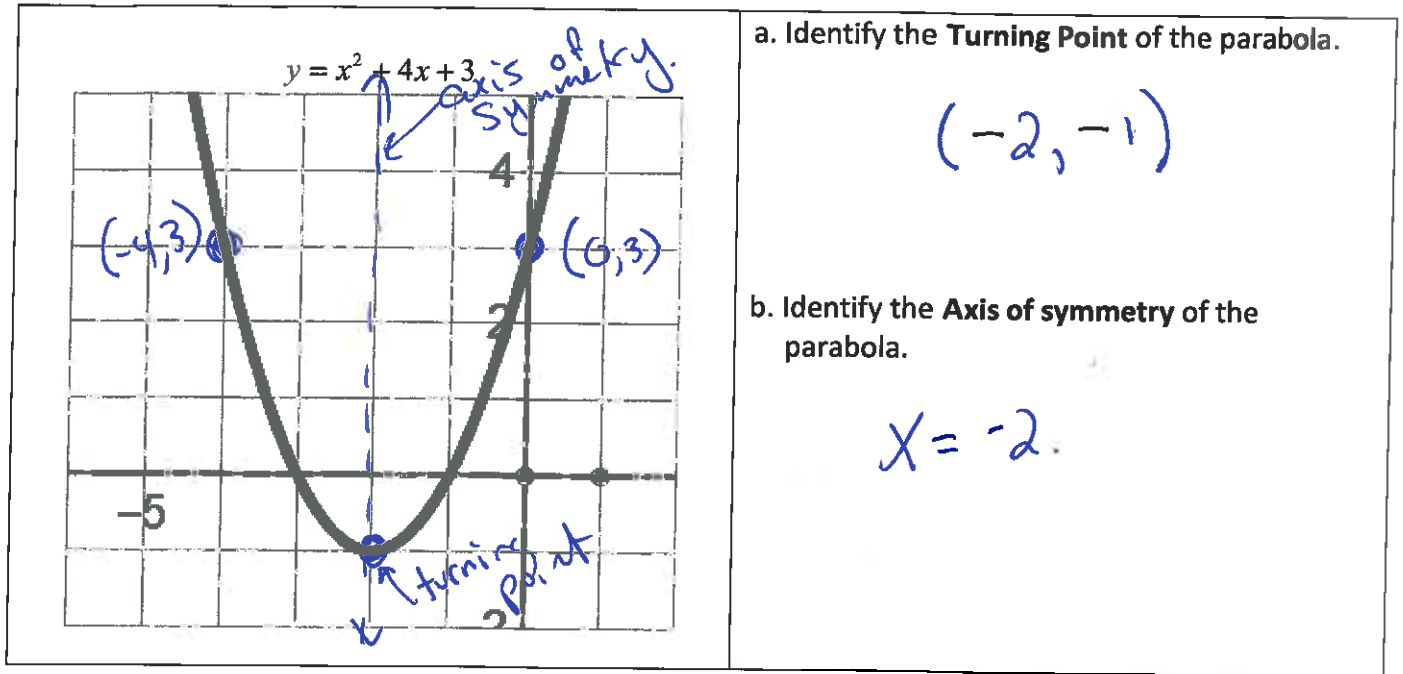
down.

Problem 3 – “The Middle of the Road”

The value “b” in a quadratic expression is referred to as the **Middle Term**.

1. The middle term (in conjunction with the leading coefficient) affects the **Turning Point** and the **Axis of Symmetry** of a parabola.

Vertex



2. Use the TRACE feature of a graphing calculator to complete the table:

Equation	a =	b =	Axis of Symmetry	Turning Point	Max or Min?
$y = x^2 + 4x + 3$	1	4	$x = -2$	$(-2, -1)$	min.
$y = 2x^2 + 4x + 3$	2	4	$x = -1$	$(-1, 1)$	min
$y = 4x^2 + 4x + 3$	4	4	$x = -\frac{1}{2}$	$(-\frac{1}{2}, 2)$	min
$y = -x^2 + 4x + 3$	-1	4	$x = 2$	$(2, 7)$	max
$y = -2x^2 + 4x + 3$	-2	4	$x = 1$	$(1, 5)$	max

3. Describe how the Axis of Symmetry can be found using the "a" and "b" values.

Divide b by -2 times a.

$$\text{ie) } x = \frac{b}{-2a}$$

4. Explain how the x-coordinate of the Turning Point is related to the Axis of Symmetry.

X-coord of turning pt is the same as the axis of symmetry.

5. Explain how the y-coordinate of the Turning Point is related to the Axis of Symmetry.

the y-coordinate is found by plugging the x-coordinate into the function.

$$\begin{aligned} \text{For example: } y &= 2x^2 + 4x + 3 & y &= 2(-1)^2 + 4(-1) + 3 \\ \text{T.P.} &= (-1, 1) & y &= 2 - 4 + 3 \\ & & y &= -2 + 3 = 1 \end{aligned}$$

6. What part of a quadratic expression determines if the Turning Point is a Maximum or Minimum?

Explain. the leading coefficient, a.

if a is positive, then the T.P. is a min. \cup

if a is negative, then the T.P. is a max. \cap

7. Determine the Axis of Symmetry and Turning Point for each quadratic by analyzing the coefficients.

State whether the Turning Point is a Maximum or Minimum.

a. $y = 3x^2 - 12x + 1$ $a=3$
 $b=-12$

$$\begin{aligned} \text{axis: } x &= \frac{-12}{(-2)(3)} \\ &= \frac{-12}{-6} \end{aligned}$$

$$\boxed{x = 2}$$

$$\begin{aligned} y &= 3(2)^2 - 12(2) + 1 \\ &= -11 \end{aligned}$$

Turning point = (2, -11)

minimum since $a > 0$

b. $y = -2x^2 + 7$ $a=-2$
 $b=0$

$$\begin{aligned} \text{axis: } x &= \frac{0}{(-2)(-2)} \\ &= \frac{0}{4} \end{aligned}$$

$$\boxed{x = 0}$$

$$\begin{aligned} y &= -2(0)^2 + 7 \\ &= 7 \end{aligned}$$

Turning point: (0, 7)

maximum
since $a < 0$

c. $y = -5x^2 - 25x$ $a=-5$
 $b=-25$

$$\begin{aligned} \text{axis: } x &= \frac{-25}{(-2)(-5)} \\ &= \frac{-25}{10} \end{aligned}$$

$$\boxed{x = -2.5}$$

$$\begin{aligned} y &= -5(-2.5)^2 - 25(-2.5) \\ &= 31.25 \end{aligned}$$

Turning pt: (-2.5, 31.25)

maximum
because $a < 0$

Problem 4 – It's "The End of the Line", So Let's "Lay Down Some Roots"

The value "c" in a quadratic expression is referred to as the **Constant Coefficient**.

The constant coefficient determines the y-intercept of the graph of a quadratic.

The x-coordinate of the x-intercepts are referred to as the **Roots** of the quadratic.

The values of "a", "b" and "c" all affect the roots of a quadratic. You will learn a method for calculating the roots algebraically in the near future. For now, we can use a calculator to estimate the roots of a quadratic.

1. Determine the y-intercept of each quadratic by analyzing the coefficients.

a. $y = x^2 - 4x - 5$

$(0, -5)$

b. $y = -3x^2 + 12x + 0$

$(0, 0)$

c. $y = x^2 + 2$

$(0, 2)$

2. Estimate the x-intercepts of the quadratic using the TRACE or INTERSECT feature of a graphing calculator and state the value of the roots.

a. $y = x^2 - 4x - 5$

$(-1, 0)$

$(5, 0)$

b. $y = -3x^2 + 12x$

$(0, 0)$

$(4, 0)$

c. $y = x^2 + 2$

no
x-intercepts.

